Exam Biological Modeling (B-B2THEC05) **30 January 2023, Educ-Alpha**

Question 1. Make a model (10 points)
Consider a closed environment containing a fixed amount of nutrients, in which two species of bacteria have been introduced. They both need a sufficient amount of two different limiting nutrients (i.e., resources like nitrogen and phosphorus) to divide. The bacteria differ in the amounts they need, they differ in their maximum birth rate, and in their expected life span. They also contain different amounts of the two limiting resources. Nutrients are released back into the environment when bacteria die. Write a natural model.

$$R_1 = K_1 - c_{11}N_1 - c_{21}N_2 \; , R_2 = K_2 - c_{12}N_1 - c_{22}N_2 \; ,$$

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = \left(b_1 \frac{R_1}{h_{11}+R_1} \frac{R_2}{h_{12}+R_2} - d_1 \right) N_1 \; , \frac{\mathrm{d}N_2}{\mathrm{d}t} = \left(b_2 \frac{R_1}{h_{21}+R_1} \frac{R_2}{h_{22}+R_2} - d_2 \right) N_2 \; ,$$

Or use a minimum function for the birth rates, e.g.,

$$b_1 \min \left(\frac{R_1}{h_{11}+R_1} \, , \frac{R_2}{h_{12}+R_2} \right) \; .$$

Question 2: Boerlijst et al., PLOS ONE 2013. (5 points)
Explain the contribution of Fig. 2a and 2b to the take home message of the paper. Hint: use the terms eigenvalue and eigenvector. Use no more than 200 words.
The take home message is that one only receives an early warning signal when the perturbations align with the eigenvector corresponding to the eigenvalue approaching zero at the saddle-node bifurcation. In Fig 2a noise is introduced on the death rate of the juveniles and in Fig. 2b this is done for the adults. Since this critical eigenvector is pointing in the direction of the juveniles we only see an early warning signal in Fig. 2a and not in 2b.

Question 3: Huisman & Weissing, Nature 1999. (5 points)
Explain the model of Eq 1–3 in your own words. Explain what the minimum function and the matrices c en K stand for. Use no more than 200 words.
Eq (1) describes the consumers with a growth rate that is limited by one of the resources, and a death rate m_i. Eq (2) describes the resources, starting with a chemostat term where S_j is concentration in the source, and $1/D$ the residence time. Eq (3) describes the growth rate, which is a minimum function of the Monod saturation terms of all the resources. The saturation constants K_{ij} describe the resource concentration at which the half maximal growth rate would be obtained for each resource.

Question 4: Berngruber et al., PLOS Pathogens 2013. (5 points)
Explain the contribution of Fig. 2 to the take home message of the paper. Use no more than 200 words.
The figure shows the predictions of the model for an infection starting at a low initial value (red lines) or at the maximum level (blue lines). In panel (a) we see the prevalence of an epidemic starting low rapidly approaches the same maximum level. In panel (b) we see that the ratio of virulent/non-virulent provirus increases initially after a low initial value, and declines immediate after a high value (prediction 1 of the model). In panel (c) we see the thing for the free virus. On top of that this ratio is much higher in the free virus than in the provirus (prediction 2 of the model).