Exam Biological Modeling (B-B2THEC05 & B-MBIMOD) **13 January 2022**

Question 1. Make a model (5 points)
A well-known example of symbiosis is the interaction between sea anemones and cleaning fish (poetsvisjes). Suppose the cleaning allows an anemone to extend its expected life span, and that the fish benefits from the anemone because they eat nutrients attached to the surface of the anemone. The removal of these attached nutrients does not harm the anemone, instead it keeps the tentacles clean, which improves the survival of the anemone (as we saw above). Write a natural model.

$$\frac{\mathrm{d}A}{\mathrm{d}t} = rA(1 - A/k) - \frac{dA}{1 + F/h} \quad \text{and} \quad \frac{\mathrm{d}F}{\mathrm{d}t} = \frac{bFA}{h + A} - \delta F\ .$$

Question 2. Make a model (10 points)
Consider a unicellular organism consisting of three cell types, i.e., two haploid and one diploid type (or equivalently a positive, a negative and a neutral cell type). All cell types have the same maximum division rate and the same death rate. Diploid (neutral) cells appear when two opposite haploid cell meet and fuse. Diploid cells are 2-fold larger and hence contain 2-fold more nutrients. After some randomly distributed time, a diploid cell will split into two haploid cells. The cells live in a closed compartment with a fixed total amount of nutrients, and their division rate is limited by the density of freely available nutrients. Because diploid cells are 2-fold larger, they need more nutrients to achieve the same division rate than haploid cells. Cells pick up nutrients when they divide and the nutrients that they contain are released in the closed environment when they die. Write a natural model.
Using a conservation equation for the nutrients we write

$$R = 1 - N_1 - N_2 - 2M\ , \quad \frac{\mathrm{d}N_1}{\mathrm{d}t} = \frac{bN_1R}{h_N + R} - dN_1 - fN_1N_2 + eM\ ,$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = \frac{bN_2R}{h_N + R} - dN_2 - fN_1N_2 + eM\ , \frac{\mathrm{d}M}{\mathrm{d}t} = \frac{bMR}{h_M + R} - dM + fN_1N_2 - eM\ ,$$

where $h_M > h_N$.

Question 3: Berngruber et al., PLOS Pathogens 2013. (5 points)
Explain in your own words why they write the $\mathrm{d}p_i/\mathrm{d}t$ and the $\mathrm{d}q_i/\mathrm{d}t$ model in Eq. (2) in addition to Eq. (1). Include an explanation for what p_i, q_i and $\phi_\bullet$ stand for. Use no more than a 100 words.
$\mathrm{d}p_i/\mathrm{d}t$ and $\mathrm{d}q_i/\mathrm{d}t$ provide the change of the frequency of the i^{th} strain of pro-virus and virus strain, respectively. Together with Eq. (1), which provides the totals, we hereby know the contribution of every strain. Moreover this provides terms illustrating the different selection pressures between the pro-virus and virus densities. The $\phi_\bullet$ stands for the average ϕ.

Question 4: Huisman & Weissing, Nature 1999. (5 points)
Explain in your own words what the matrices C en K in the "Methods" section stand for. Include an explanation for what a particular matrix element, K_{ji}, and a particular matrix element, C_{ji}, stand for. Use no more than a 100 words.
The matrix K collects all the saturation constants from Eq. (3). K_{ji} defines the resource density j that species i need to achieve a half-maximal growth rate. The matrix C collects all the resource 'contents', i.e., the amount of resource j contained in a single cell of species i is given by the element C_{ji}.

Question 5: Boerlijst et al., PLOS ONE 2013. (5 points)
Explain why in Fig. 2a the coefficient of variation increases for the juveniles while in Fig. 2b this does not happen. Use the terms eigenvalue and eigenvector, but use no more than a 100 words.
In Fig 2a noise is introduced on the death rate of the juvelines and in Fig. 2b this is done for the adults. The eigenvector corresponding to the eigenvalue that is going through zero at a predator mortality $\mu_P = 0.553$ is pointing in the direction of the juveniles. Perturbations recover at the slow rate determined by this small eigenvalue.