Name __________________________

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Note, if a question has several sections, please write answers for each section separately.
One parametric study

I. Study the following system of ODEs using content /grind:

\[
\dot{x} = \frac{1}{0.05 + x} - y \quad x \geq 0
\]

\[
\dot{y} = -3. * y - x + k \quad y \geq 0
\]

Study this model for \(0 < k < 6\), i.e.:

1. Which bifurcation/bifurcations do we have here.
2. Draw the bifurcation diagram on \((x, k)\) plane.
3. How many regions of different behavior do we have here?
4. Draw phase portraits in each of these regions
5. Show attractor/attractors and schematically show basin/basins of attraction
Two parametric study

II. Consider the Holling-Tanner model for predator-prey interaction: Study the following system of ODEs using content/grind:

\[
\begin{align*}
\dot{x} &= rx(1-x) - \frac{wxy}{x+y} \\
\dot{y} &= sy(1-bx)
\end{align*}
\]

Put \(r = w = D = b = 1\). Perform two parametric study of this system using content/grind in the following parameter range:

\[0. < s < 1. \quad 1. < K < 20\]
III. Find analytically stability of equilibrium point $x = 0, y = 0$ of the following system of ODEs:

\[
\begin{align*}
\frac{dx}{dt} &= x \cdot y + y^2 \\
\frac{dy}{dt} &= -y - x^2
\end{align*}
\]
IV. Find analytically the point of flip bifurcation of the following map for $B=4$ and $a > 0$.

$$x_{n+1} = 3ax_n - Bx^2$$

Do the same for arbitrary $B > 0$. 
**Extra I.** Find analytically location of the line of the Hopf bifurcation, for the following normal form for the Bogdanov-Takens bifurcation. Show it on the \((\beta_1, \beta_2)\) plane.

* What is the type of this Hopf bifurcation.

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -\beta_1 + \beta_2 y + x^2 - x y
\end{align*}
\]
Extra II. The following system of ODEs was used to describe a biological system:

\[
\begin{align*}
\frac{dx}{dt} &= -x + 2y + x^2 \\
\frac{dy}{dt} &= 3x - y - 3x^3 + 1.5xy - cx
\end{align*}
\]

Study this system using content for $-1 < c < 3$; Draw bifurcation diagram and phase portraits at each of the regions of different qualitative behavior.